Standard 2-2: The student will demonstrate through the mathematical processes an understanding of the base-ten numeration system; place values; and accurate, efficient, and generalizable methods of adding and subtracting whole numbers.

Indicator 2-2.1

Generate estimation strategies to determine the approximate number of objects in a set of no more than 1,000 objects.

Continuum of Knowledge

In first grade, students used estimation to determine the approximate number of objects in a set of 20 to 100 objects. (1-2.2)

In second grade, students generate estimation strategies to determine the approximate number of objects in a set of no more than 1,000 objects. (2-2.1) Students analyze the magnitude of digits through 9,999 on the basis of their place value. (2-2.1)

In third grade, students will compare whole-number quantities through 999,999 by using the terms is less than, is greater than, and is equal to and the symbols <, > and = . (3-2.1)

Taxonomy Level

Cognitive Dimension: Create
Knowledge Dimension: Conceptual

Key Concepts

- Estimate/estimation
- Object/set
- thousand

Instructional Guidelines

For this indicator, it is essential for students to:

- Create their own strategies to estimate the approximate number of objects in a set.
- Develop a sense of number and a sense of magnitude, the size of the number. (Students need to visualize what the number looks like to have a sense of the magnitude of the number. Ex. 345 would have 3 flats, 4 rods and 5 units or 300 + 40 + 5.)
For this indicator, it is **not essential** for students to:

- To estimate the number of objects in a set with more than 1000 objects.

**Student Misconceptions/Errors**

- Students have a tendency to count the number of objects that can be seen and totally disregard the remaining objects in the container. Students should have opportunities to count the number of objects in a container and discuss how many were hidden.

**Instructional Resources and Strategies**

- Students are to generate the strategies rather than the teacher giving suggestions. Teachers ask guiding questions to mediate the student’s thinking as they move through the process of generating a strategy. “Often you can present a problem and have students suggest solutions or strategies. There suggestions will not solve the problem for others because students must still work out the solution an explanation…brainstorming will likely produce a variety of approaches, resulting in more profitable solutions by more students.” Van De Walle 2006 p 43
- Teachers should provide multiple “hands on” experiences with “real world” containers and objects to help students make the connections that large objects take up more space and would have a lower estimation, and smaller objects would take up less space and have a higher estimation.
- Students have opportunities to select an estimate when the actual container is in front of them, to create benchmarks and then move to pictures of containers.

The students must estimate the value of a set of base 10 materials and indicate on an interactive number line. The sets displayed range from 100 to 1000. It is an appropriate whole group tool for tutorial as well as practice on individual computers. Since there is a timer and the program keeps score for the player, it may be used as a competitive game with students. The teacher can look at the screen and determine which estimates are in range and out of range. Since students must calculate the value of the base ten materials, skip counting by hundreds, tens, and ones as well as mental computation is reinforced.
- A meaningful connection exists between estimation/magnitude of number and place value. For example, in small groups, students are asked to estimate the number of objects in a large jar. They may dump out the jar and count the objects inside by using a place value organization of tens, hundreds, and thousands on the floor or table. The objects may be counted and placed into piles of tens until there are 10 tens, at which point those objects may be combined into a pile of one hundred. The objects continue to be counted until the group finishes. Students should represent their thinking
by writing and drawing diagrams in math journals about each strategy used to count and group the objects and the value of each digit in the actual amount.

Assessment Guidelines

The objective of this indicator is to *generate*, which is in the “create conceptual” knowledge of the Revised Taxonomy table. Conceptual knowledge is not bound by specific examples; therefore students should *generate* several strategies to determine the approximate number in a set using a wide variety of examples. The learning progression to *generate* requires students to *recognize* the number of objects in smaller sets. They must *analyze* the size of the objects compared to the size of the container and *explain* their reasoning (2-1.3). As the sets increase, the students should generate strategies to estimate sets up to 1,000 objects. They will *exchange* mathematical ideas and *generate* conjectures with classmates. (2-1.2)
Standard 2-2: The student will demonstrate through the mathematical processes an understanding of the base-ten numeration system; place values; and accurate, efficient, and generalizable methods of adding and subtracting whole numbers.

Indicator 2-2.2

Represent quantities in word form through twenty.

Continuum of Knowledge

In kindergarten, students translate between numeral and quantity through 31. (K-2.2)

In first grade, students represent quantities in word form through ten. (1-2.3)

In second grade, students represent quantities in word form through twenty. (2-2.2), and represent multiples of ten in word form through ninety. (2-2.3)

In third grade, students will represent in word form whole numbers through nine hundred ninety-nine thousand. (3-2.2)

Taxonomy Level

Cognitive Dimension: Understand
Knowledge Dimension: Factual

Key Concepts

- Eleven
- Twelve
- Thirteen
- Fourteen
- Fifteen
- Sixteen
- Seventeen
- Eighteen
- Nineteen
- Twenty

Instructional Guidelines

For this indicator, it is essential for students to:

- Understand 1 – 1 correspondence to represent quantities through 20.
- Read and write words through 20.
For this indicator, it is **not essential** for students to:

- To spell number words.

**Student Misconceptions/Errors**

- Students often confuse twelve and twenty
- “In first and second grade, children need to connect the base ten concepts with the oral names they have used many times. They know the words but have not thought of them in terms of tens and ones. ...Use base ten language paired with standard language. Emphasize the teens as exceptions. Acknowledge that they are formed “backward” and do not fit the pattern.” For example: 17..71.(Van De Walle 2004 p188)

**Instructional Resources and Strategies**

- Connect place value... one ten and one =11 = eleven
- One ten and two = 12 = twelve

**Assessment Guidelines**

The objective of this indicator is to **represent**, which is the “understand factual” cell of the Revised Taxonomy table. Factual knowledge is bound by specific examples; therefore students should **recognize** the vocabulary and be able to represent quantities in word form. The learning progression to **represent** through twenty requires that they **recognize** quantities through ten and **explain** their reasoning (2-1.3).
**Standard 2-2**: The student will demonstrate through the mathematical processes an understanding of the base-ten numeration system; place values; and accurate, efficient, and generalizable methods of adding and subtracting whole numbers.

**Indicator 2-2.3**

Represent multiples of ten in word form through ninety.

**Continuum of Knowledge**

In first grade, students represented quantities in word form up to ten. (1-2.3)

Second grade students will represent quantities in word form up to twenty. (2-2.2) and represent the multiples of ten in word form through ninety. (2-2.3)

In third grade, students will represent in word form whole numbers through nine hundred ninety-nine thousand. (3-2.2)

**Taxonomy Level**

Cognitive Dimension: Understand
Knowledge Dimension: Factual

**Key Concepts**

- Ten
- Twenty
- Thirty
- Forty
- Fifty
- Sixty
- Seventy
- Eighty
- Ninety

**Instructional Guidelines**

For this indicator, it is **essential** for students to:

- Count to ten
- Make groups of ten
- Count by tens
- Recognize multiples of ten in numeric form
- Recognize and recall multiples of ten in word form
For this indicator, it is **not essential** for students to:

- Spell multiples of ten in word form
- Recognize multiples of ten greater than ninety

**Student Misconceptions/Errors**

None noted

**Instructional Resources and Strategies**

- Use a 10 X 10 array of dots on the overhead projector. Cover up all but two rows. How many tens? Two tens is called twenty. Show another row. Three tens is called thirty. Four tens is called forty. Five tens should have been called fivety rather than fifty. Continue the process until you reach ninety. Slide the cover up and down and continue asking how many tens and the name for each. (taken from Van De Walle K-3)

**Assessment Guidelines**

The objective of this indicator is to represent, which is the “understand factual” cell of the Revised Taxonomy table. Factual knowledge is bound by specific examples. Therefore, students should represent multiples of ten in word form through ninety. The learning progression to represent requires that students recognize the word form and connect the quantity to the word form and explain their reasoning. (2-1.3)
Standard 2-2: The student will demonstrate through the mathematical processes an understanding of the base-ten numeration system; place values; and accurate, efficient, and generalizable methods of adding and subtracting whole numbers.

Indicator 2-2.4

Compare whole number quantities through 9,999 by using the terms is less than, is greater than, and is equal to and the symbols <, >, and =.

Continuum of Knowledge

In first grade, compare whole-number quantities through 100 by using the terms is greater than, is less than, and is equal to. (1-2.5) Students will analyze the magnitude of digits through 999 on the basis of their place value. (1-2.9)

In second grade, students compare whole-number quantities through 9,999 by using the terms is greater than, is less than, and is equal to and the symbols <, >, and =. (2-2.4) Students will analyze the magnitude of digits through 9,999 on the basis of their place value. (2-2.10)

In third grade, students compare whole number quantities through 999,999 by using the terms is less than, is greater than, and is equal to and the symbols <, >, and =. (3-2.1) Students will also analyze the magnitude of digits through 999,999 on the basis of their place value. (3-2.12)

Taxonomy Level

Cognitive Dimension: Understand
Knowledge Dimension: Conceptual

Key Concepts

- Compare
- Is greater than
- Is less than
- Is equal to
- Place value

Math Notation/Symbols

- <
- >
- =

Instructional Guidelines

For this indicator, it is essential for students to:
Numbers and Operations  
Second Grade

- Recognize the place value of digits through 9,999
- Compare the place value of digits through 9,999
- Recognize mathematical symbols <, >, and = and their meanings

For this indicator, it is **not essential** for students to:

- Compare whole number quantities greater than 9,999

**Student Misconceptions/Errors**

Though the concept of less is logically equivalent to the concept of more, the word less proves to be more difficult for children than more. A possible explanation is that children have many opportunities to use the word more but have limited exposure to the word less. To help children with the concept of less, frequently pair it with the word more and make a conscious effort to ask “which is less?” questions as well as “which is more?” (Van De Walle 2006)

**Instructional Resources and Strategies**

- National Library of Virtual Manipulatives
- Play Compare with partners using base ten pieces, a cube with the words unit- rod- flat on the faces and a number cube. Roll the 2 cubes, set out the collection and then compare. Write a number sentence to match the collection.
- Create a sheet with three columns, number cards 0 – 9 or number cubes. Draw 3 cards or roll the cube 3 times and make the largest number possible or the smallest number.

**Assessment Guidelines**

The objective of this indicator is to compare, which is the “understand conceptual” knowledge cell of the Revised Taxonomy Table. Conceptual knowledge is not bound by specific examples; therefore students should compare hundreds to hundreds, tens to tens, ones to ones and combinations of hundreds, tens, and ones

The learning progression to **compare** requires students to **recognize** the place value of digits through 9,999, **compare** the place value of digits through 9,999, **recognize** mathematical symbols <, >, and = and their meanings. Throughout the learning experience, students **analyze** place value patterns (2-1.4) and **generate** conjectures and exchange mathematical ideas about which symbol is appropriate.(2-1.2). For example: 4 flats > 4 rods > 4 units or 4 tens > 4 ones. Students **use** a variety of forms of mathematical communication such as words, symbols, numbers, and pictures. (2-1.6)

**Standard 2-2:** The student will demonstrate through the mathematical processes an understanding of the base-ten numeration system; place values; and accurate,
Indicator 2-2.5

Interpret models of equal grouping (multiplication) as repeated addition and arrays.

Continuum of Knowledge

In second grade, students interpret models of equal grouping as repeated addition and arrays. (2-2.5)

In third grade, students recall basic multiplication facts through 12 × 12 and the corresponding division facts. (3-2.7)

Taxonomy Level

Cognitive Dimension: Understand
Knowledge Dimension: Conceptual

Key Concepts

- Repeated addition
- Arrays
- Addends
- Equal grouping/sets
- Vertical columns
- Horizontal rows

Instructional Guidelines

For this indicator, it is essential for students to:

- Use concrete objects in arrays and then pictures of arrays and equal groupings that show repeated addition.
- Act out number stories that illustrate multiplication as repeated addition and represent the same story by creating arrays.

For this indicator, it is not essential for students to:

- Be introduced to actual multiplication.
- Memorize multiplication facts.

Student Misconceptions/Errors
None noted

**Instructional Resources and Strategies**

Teachers should provide multiple experiences with real life examples to make the connection that repeated equal groupings can also be represented with arrays. Teachers should use the terms vertical columns and horizontal rows when describing the arrays. Student could use their shoes to show that 1 person with 2 shoes has 2 shoes and the resulting array of 1 by 2. Two students with 2 shoes each would have $2 + 2 = 4$ shoes, or two groups of two, resulting in a 2 by 2 array, etc. The same could be done with 3, 4, 5, groups as well.

**Assessment Guidelines**

The objective of this indicator is to interpret, which is in the “understand conceptual” cell of the Revised Taxonomy table. Conceptual knowledge is not bound by specific examples; therefore students should translate, represent, clarify, and paraphrase models of equal grouping. The learning progression to represent requires students to recognize that repeated addition is adding the same number and that arrays are pictorial representations of repeated addition. Students then construct arrays to represent repeated addition. Given array models, students analyze for patterns of repeated addition and generalize mathematical concepts of equal grouping as repeated addition and arrays. (2-1.4, 2-1.5)
Standard 2-2: The student will demonstrate through the mathematical processes an understanding of the base-ten numeration system; place values; and accurate, efficient, and generalizable methods of adding and subtracting whole numbers.

Indicator 2-2.6

Interpret models of sharing equally (division) as repeated subtraction and arrays.

Continuum of Knowledge

In second grade, students interpret models of sharing equally as repeated subtraction and arrays. (2-2.6)

In third grade, students recall basic multiplication facts through $12 \times 12$ and the corresponding division facts. (3-2.7)

Taxonomy Level

Cognitive Dimension: Understand
Knowledge Dimension: Conceptual

Key Concepts

- Repeated subtraction
- Arrays
- Equal sharing/sets
- Vertical columns
- Horizontal rows

Instructional Guidelines

For this indicator, it is essential for students to:

- Use concrete objects in arrays and then pictures of arrays and sharing equally that show repeated subtraction.
- Act out number stories that illustrate division as repeated subtraction and represent the same story by creating arrays.

For this indicator, it is not essential for students to:

- Memorize division facts.
- Be exposed to remainders.
- Be introduced to actual division.
Student Misconceptions/Errors

None noted

Instructional Resources and Strategies

Teachers should provide multiple experiences with real life examples to make the connection that sharing equally can also be represented with arrays. Teachers should use the terms vertical columns and horizontal rows when describing the arrays. In *The Doorbell Rang* twelve cookies are shared by friends. If there were two friends the cookies would be shared as a 2 by 6 array. If three friends shared then a 3 by 4 array would result.

Assessment Guidelines

The objective of this indicator is to interpret, which is in the “understand conceptual” cell of the Revised Taxonomy table. Conceptual knowledge is not bound by specific examples; therefore students should translate, represent, clarify, and paraphrase models of equal grouping. The learning progression to represent requires students to recognize that repeated subtraction is subtracting the same number and that arrays are pictorial representations of repeated subtraction. Students then construct arrays to represent repeated subtraction. Given array models, students analyze for patterns of repeated subtraction and generalize mathematical concepts of sharing equally as repeated subtraction and arrays. (2-1.4, 2-1.5)
Standard 2-2: The student will demonstrate through the mathematical processes an understanding of the base-ten numeration system; place values; and accurate, efficient, and generalizable methods of adding and subtracting whole numbers.

Indicator 2-2.7

Generate strategies to add and subtract pairs of two-digit whole numbers with regrouping.

Continuum of Knowledge

In kindergarten, students represented simple joining and separating situations through 10. (K-2.4) and developed an understanding that addition results in increase and subtraction results in decrease. (K-2.5) In first grade, students summarized the inverse relationships between addition and subtraction. (1-2.7) Students generated strategies to add and subtract without regrouping two-digit numbers. (1-2.7) They have recall of basic addition facts through 9 + 9 and corresponding subtraction facts. (1-2.6)

In second grade, students generate strategies to add and subtract pairs of two-digit whole numbers with regrouping. (2-2.7)

In third grade, students apply an algorithm to add and subtract whole numbers fluently. (3-2.3)

Taxonomy Level

Cognitive Dimension: Create
Knowledge Dimension: Conceptual

Key Concepts

- Add
- Subtract
- Strategies
- Two-digit
- Place value
- Tens
- ones

Instructional Guidelines

For this indicator, it is **essential** for students to:

- To make the connection between the concrete/pictorial models and the symbolic form when working with two-digit numbers that do not involve regrouping.
• Create their own strategies using concrete/pictorial models when working with two-digit numbers that involve regrouping.

For this indicator, it is **not essential** for students to:

• Learn a traditional algorithm involving regrouping.

**Student Misconceptions/Errors**

Students may think there is only one way to add or subtract.

**Instructional Resources and Strategies**

Students should be given problem solving situations that require them to regroup, second grade students should generate their own strategies using their knowledge of place value and basic addition and subtraction facts. The emphasis is on students being able to generate a strategy that works and showing the strategy using pictorial or concrete models. With regard to regrouping, second grade students should NOT be required to symbolically (numbers only) solve addition and subtraction problems that require regrouping. When generating strategies students should be able to select the most efficient method to solve a problem and to justify the reasonableness of their answers. The expectation is they will *generate strategies* of their own choosing to add and subtract the quantities instead of learning a teacher-directed algorithm. The part-part-whole model should be emphasized to connect the concepts.

**Assessment Guidelines**

The objective of this indicator is to *generate*, which is in the “create conceptual” cell of the Revised Taxonomy table. Conceptual knowledge is not bound by specific examples; therefore students should *generate* several strategies to add and subtract pairs of two-digit numbers using a variety of examples.

The learning progression to *generate* requires students to *recall* basic addition facts through 9 + 9 and corresponding subtraction facts and to add and subtract 2-digit numbers without regrouping. Students will *generate* conjectures and exchange mathematical ideas when working with 2-digit numbers that involve regrouping. (2-1.2) Students *explain* and *justify* their ideas using multiple informal and concrete and pictorial representations to convey mathematical ideas. (2-1.3, 2-1.8)
Standard 2-2: The student will demonstrate through the mathematical processes an understanding of the base-ten numeration system; place values; and accurate, efficient, and generalizable methods of adding and subtracting whole numbers.

Indicator 2-2.8

Generate addition and subtraction strategies to find missing addends and subtrahends in number combinations through 20.

Continuum of Knowledge

In second grade, students generate addition and subtraction strategies to find missing addends and subtrahends in number combinations through 20.

Taxonomy Level

Cognitive Dimension: Create
Knowledge Dimension: Conceptual

Key Concepts

- Addends
- Subtrahends
- Combinations
- equals

Instructional Guidelines

For this indicator, it is essential for students to:

- Understand the meaning of the equal sign, equally balanced
- Create their own strategies

For this indicator, it is not essential for students to:

None noted

Student Misconceptions/Errors

- Students see the equal sign to mean “perform an operation” rather than a sign of equivalence between two parts of an equation. As a result, when presented with a problem such as 4 + ___ = 7, students use the numbers to perform an operation rather than focusing on what should be added to 4 to equal 7.
Instructional Resources and Strategies

Students must be very flexible with addition and subtraction up to 20 and be able to demonstrate inverse relationships. To do so, they should have experiences using concrete and pictorial models and connect the models to writing in numerical sentences, and then finding missing addends and subtrahends in combinations to 20. A number balance is a visual and hands-on tool that demonstrates concretely strategies to figure out the missing parts in an equation by manipulating the masses on the balance’s pegs. Being able to generate strategies to find missing addends sets the stage for students understanding of the concept of equivalence.

Assessment Guidelines

The objective of this indicator is to generate, which is in the “create conceptual” cell of the Revised Taxonomy table. Conceptual knowledge is not bound by specific examples; therefore students should generate addition and subtraction strategies to find missing addends and subtrahends in number combinations through 20. The learning progression to generate requires students to recall basic addition facts through 9 + 9 and their corresponding subtraction facts. Students will generate conjectures and exchange mathematical ideas to find missing addends and subtrahends in number combinations through 20. (2-1.2) For example, using a number balance. Students explain and justify their ideas using multiple informal concrete and pictorial representations to convey mathematical ideas. (2-1.8)
**Standard 2-2:** The student will demonstrate through the mathematical processes an understanding of the base-ten numeration system; place values; and accurate, efficient, and generalizable methods of adding and subtracting whole numbers.

**Indicator 2-2.9**

Generate strategies to round numbers through 90 to the nearest 10.

**Continuum of Knowledge**

In second grade, students generate strategies to round numbers through 90 to the nearest 10. (2-2.9)

In third grade, students apply procedures to round any whole number to the nearest 10, 100, or 1000. (3-2.4)

**Taxonomy Level**

Cognitive Dimension: Create  
Knowledge Dimension: Understand

**Key Concepts**

- Estimate  
- Round  
- Whole number

**Instructional Guidelines**

For this indicator, it is **essential** for students to:

- Have experience with a number line as well as concrete objects.  
- Understand that they may want to round numbers to make mental calculations easier.  
- Create their own strategies

For this indicator, it is **not essential** for students to:

- apply a formal procedure to round numbers to the nearest 10, 100, or 1000.

**Student Misconceptions/Errors**

None noted
**Instructional Resources and Strategies**

To round a number simply means to substitute a nice number that is close so that some computation can be done more easily. A number line with nice numbers highlighted can be useful in helping children select near nice numbers. Indicate a number above the line that you want to round. Discuss the marks (nice numbers) that are close.

**Assessment Guidelines**

The objective of this indicator is to generate, which is in the “create conceptual” cell of the Revised Taxonomy table. Conceptual knowledge is not bound by specific examples: therefore students should generate several strategies to round numbers through 90 to the nearest 10. The learning progression to generate requires students to identify the place value of two-digit numbers. Students compare two-digit numbers to the benchmark multiples of 10. For example, 54 compared to 50 and 60. Students generate conjectures and exchange mathematical ideas for rounding numbers to the nearest 10. (2-1.2) students should explain and justify their thinking. (2-1.)
**Standard 2-2:** The student will demonstrate through the mathematical processes an understanding of the base-ten numeration system; place values; and accurate, efficient, and generalizable methods of adding and subtracting whole numbers.

**Indicator 2-2.10**

Analyze the magnitude of digits through 9,999 on the basis of their place value.

**Continuum of Knowledge**

In kindergarten, the students analyze the magnitude of digits through 99 on the basis of their place values. (K-2.6). In first grade, students analyze the magnitude of digits through 999 on the basis of their place values. (1-2.9)

In second grade, students analyze the magnitude of digits through 9,999 on the basis of their place values. (2-2.10)

In third grade students, analyze the magnitude of digits through 999,999 on the basis of their place values. (3-2.12)

**Taxonomy Level**

Cognitive Dimension: Analyze
Knowledge Dimension: Conceptual

**Key Concepts**

- digits
- Place value
- Value
- Units/ones
- Tens/rods
- Hundreds/flats
- Thousands/cube
- Standard form
- Expanded form
- Compose/decompose
- Equivalent forms

**Instructional Guidelines**

For this indicator, it is **essential** for students to:
• Analyze and break apart a whole number into its parts and make the connection that place value of numbers is based on Ten. Ten units/ones is needed to make a ten, ten tens is needed to make a hundred, ten hundreds is needed to make a thousand.
• To write numbers in expanded form and standard form.
• Recognize equivalent representations by composing (ex. putting a number together from parts, standard form) and decomposing (ex. breaking apart a number, expanded form) whole numbers.

For this indicator, it is not essential for students to:

• Analyze the magnitude of digits up to ten thousand.

Student Misconceptions/Errors

None noted

Instructional Resources and Strategies

Use multiple names for place value to give the learner a visual image of the number. Example: units/ones, tens/rods, hundreds/flats. Students should continue to use concrete and pictorial materials to represent the magnitude of numbers.

Assessment Guidelines

The objective of this indicator is to analyze, which is in the "analyze conceptual” cell of the Revised Taxonomy table. Conceptual knowledge is not bound by specific examples; therefore students should analyze the magnitude of digits on the basis of their place value. The learning progression to analyze requires students to recall the value of the digits, and apply that understanding to larger numbers. Students should be able to compare the magnitude of digits on the basis of their place value. Students explain and justify their thinking and use multiple informal representations to convey their mathematical understanding of this idea. (2-1.3, 2-1.8) For example, 9 rods = 90.
Standard 2-3: The student will demonstrate through the mathematical processes an understanding of numeric patterns and quantitative change.

Indicator 2-3.1

Analyze numeric patterns in skip counting that uses the numerals 1 through 10.

Continuum of Knowledge

In kindergarten, students identify simple patterns. (K-3.1) Students analyze simple repeating and growing relationships to extend patterns. (K-3.2) Students translate simple repeating and growing patterns into rules. (K-3.3). In first grade, students analyze numeric patterns in addition and subtraction to develop strategies for acquiring basic facts. (1-3.1) Students analyze numeric relationships to complete and extend simple patterns. (1-3.4)

In second grade, students analyze numeric patterns in skip counting that uses numerals 1 through 10. (2-3.1)

In third grade, students create numeric patterns that involve whole-number operations. (3-3.1) Students apply procedures to find missing numbers in numeric patterns that involve whole-number operations. (3-3.2)

Taxonomy Level

Cognitive Dimension: Analyze
Knowledge Dimension: Conceptual

Key Concepts

• Pattern
• Skip counting
• Numeric

Instructional Guidelines

For this indicator, it is essential for students to:

• Skip count using numerals 1 – 10
• Have a knowledge of pattern organization

For this indicator, it is not essential for students to:

• Create numeric patterns
Algebra  
Grade Two

Student Misconceptions/Errors

None noted

Instructional Resources and Strategies

Students in second grade build on prior knowledge of skip counting by starting with any number 1-10 and skip counting through 100. Students should analyze the relationship between the numbers created by skip counting with any number. While a pattern can be identified by merely skip counting, the mathematics in this indicator goes deeper. Students should analyze the numeric relationship between numbers created as a result of the skip counting. For example, when skip counting by 3, students should examine the relationship between 3, 6, 9, etc. the difference between each element is also a pattern— a difference of 3 each time. This seems obvious to the adult learner, but it is a numeric relationship that must be analyzed by students in order to develop number sense and comprehension.

Assessment Guidelines

The objective of this indicator is to analyze, which is in the "analyze conceptual" cell of the Revised Taxonomy table. Conceptual knowledge is not bound by specific examples: therefore students should analyze numeric patterns in skip counting that uses the numerals 1 through 10. The learning progression to analyze requires students to recall how to skip count through 10 and recognize skip counting patterns and differentiate patterns by reasoning systematically (2-1.4).
Standard 2-3: The student will demonstrate through the mathematical processes an understanding of numeric patterns and quantitative change.

Indicator 2-3.2

Translate patterns into rules for simple multiples.

Continuum of Knowledge

In kindergarten, students translate simple repeating and growing patterns into rules. (K-3.3)

In first grade, students translate patterns into rules for simple addition and subtraction. (1-3.2)

In second grade, students translate patterns into rules for simple multiples. (2-3.2)

In third grade, students create numeric patterns that involve whole number operations. (3-3.1)

Taxonomy Level

Cognitive Dimension: Understand
Knowledge Dimension: Conceptual

Key Concepts

- Multiples
- Patterns
- Rules

Instructional Guidelines

For this indicator, it is essential for students to:

- Skip count using numerals 1 – 10
- Apply skills and knowledge about translating patterns into rules for simple addition and subtraction.

For this indicator, it is not essential for students to:

- Understand the concept of multiplication
**Student Misconceptions/Errors**

Using multiples is not to be confused with multiplication facts.

**Instructional Resources and Strategies**

Students notice how numeric patterns repeat via addition which leads to multiples, not multiplication.

**Assessment Guidelines**

The objective of this indicator is to understand, which is in the “understand conceptual” cell of the Revised Taxonomy table. Conceptual knowledge is not bound by specific examples: therefore students should translate patterns into rules for simple multiples. The learning progression to understand requires students to recognize skip counting patterns and differentiate patterns by reasoning systematically (2-1.4). The students will explain and justify the rules for simple multiples. (2-1.3)
**Standard 2-3:** The student will demonstrate through the mathematical processes an understanding of numeric patterns and quantitative change.

**Indicator 2-3.3**

Analyze relationships to complete and extend growing and repeating patterns involving numeric, symbols and objects.

**Continuum of Knowledge**

In kindergarten, students analyze simple repeating and growing relationships to extend patterns. (K-3.2)

In first grade, students analyze numbers relationships to complete and extend simple patterns. (1-3.4)

In second grade, students analyze relationships to complete and extend growing and repeating patterns involving numbers, symbols and objects. (2-3.3)

In third grade, students use symbols to represent an unknown quantity in a simple addition, subtraction, or multiplication equation. (3-3.3)

**Taxonomy Level**

Cognitive Dimension: Analyze
Knowledge Dimension: Conceptual

**Key Concepts**

- Growing pattern
- Repeating pattern
- Extend
- Core (the part that repeats)
- Element (the numbers, symbols, objects)
- Analyze

**Instructional Guidelines**

For this indicator, it is **essential** for students to:

- Understand growing and repeating patterns
- Complete and extend patterns
- Recognize even and odd patterns
For this indicator, it is **not essential** for students to:

None noted

**Student Misconceptions/Errors**

Students often think that all patterns repeat after the last element shown.

**Instructional Resources and Strategies**

In a repeating pattern, the core must be repeated at least twice. Growing patterns consist of a series of separate steps, with each new step related to the previous one according to the pattern. For most repeating patterns, the elements of the pattern can be numbered 1, 2, 3, and so on. Before students begin to extend the pattern, have them predict exactly what element will be in number 15 position or the number 27 position. Students should be required to provide a reason for their prediction, preferably in writing. Students should then extend the pattern and check their prediction. If their prediction is incorrect, have them examine their reasoning and try to figure out why the prediction was off. Van de Walle (2006) Show students the first three or four steps in a pattern. Provide them with appropriate materials and grid paper, have them extend the patterns recording each step, and explain why their extension indeed follows the pattern. Van de Walle (2006) Square numbers and triangular numbers are examples that could be used.

**Assessment Guidelines**

The objective of this indicator is to **analyze**, which is in the “analyze conceptual” cell of the Revised Taxonomy table. Conceptual knowledge is not bound by specific examples: therefore students should **analyze** relationships to complete and extend growing and repeating patterns involving numbers, symbols, and objects. The learning progression to **analyze** requires students to **identify** a growing or repeating pattern, **recall** and **recognize** the pattern, then **analyze** patterns by reasoning systematically (2-1.4). Students should **explain** and **justify** their answers (2-1.3) using a variety of forms of communication (2-1.6).
Standard 2-3: The student will demonstrate through the mathematical processes an understanding of numeric patterns and quantitative change.

Indicator 2-3.4

Identify quantitative and qualitative change over time.

Continuum of Knowledge

In first grade, students classify change over time as quantitative or qualitative. (1-3.3)

In second grade, students identify quantitative and qualitative change over time. (2-3.4)

In third grade, students illustrate situations that show change over time as increasing. (3-3.4)

Taxonomy Level

Cognitive Dimension: Understand
Knowledge Dimension: Conceptual

Key Concepts

- Quantitative
- Qualitative

Instructional Guidelines

For this indicator, it is essential for students to:

- Compare examples of quantitative and qualitative change.
- Understand the difference between quantitative and qualitative change.
- Use the terms qualitative and quantitative

For this indicator, it is not essential for students to:

- Illustrate situations that show change over time as increasing.

Student Misconceptions/Errors

None noted

Instructional Resources and Strategies
Students will identify quantitative (number) and qualitative (attribute) changes. An example of quantitative change would be growing 4 inches, while growing taller would be the qualitative change.

An example of a qualitative question students should be able to answer is, “Taylor is taller this year than last year. Identify the type of change over time.” As students analyze this qualitative change over time, students should be able to say that the time is one year and the change is Taylor growing taller.

An example of a quantitative question students should be able to answer is, “Taylor is 3 inches taller this year than last year. Identify the type of change over time.” As students analyze this quantitative change over time, students should be able to say that the time is one year and the change is three inches taller. Other examples that could be used would be popsicles melting or being eaten, pencils used during the day, or Twizzlers being eaten.

**Step by Step** by Bruce McMillian
Pictures show a little boy as he grows from being a crawler to a walker. It spans four months to fourteen months—thus quantitative and qualitative illustrations. Before and after pictures can be used to identify change over time. Example: melting snow.

**Assessment Guidelines**

The objective of this indicator is to identify, which is in the “understand conceptual” cell of the Revised Taxonomy table. Conceptual knowledge is not bound by specific examples: therefore students should compare quantitative and qualitative change over time. The learning progression to identify requires students to analyze situations and determine if change has occurred. They generate conjectures and exchange mathematical ideas (2-1.2) about their observations. They use that information to identify the change as quantitative and qualitative. Students also give examples of situations that show a change in quality and a change in quantity.
Standard 2-3: The student will demonstrate through the mathematical processes an understanding of numeric patterns and quantitative change.

Indicator 2-3.5

Analyze quantitative and qualitative change over time.

Continuum of Knowledge

In first grade, students classify change over time as quantitative or qualitative. (1-3.6)

In second grade, students analyze quantitative and qualitative change over time. (2-3.5)

In third grade, students illustrate situations that show change over time as increasing. (3-3.4)

Taxonomy Level

Cognitive Dimension: Analyze
Knowledge Dimension: Conceptual

Key Concepts

- Quantitative
- Qualitative
- Analyze

Instructional Guidelines

For this indicator, it is essential for students to:

- understand the difference between quantitative and qualitative change over time.

For this indicator, it is not essential for students to:

- illustrate situations that show change over time as increasing.

Student Misconceptions/Errors

None noted
**Instructional Resources and Strategies**

An example of a qualitative question students should be able to answer is, “Taylor is taller this year than last year. Identify the type of change over time.” As students analyze this qualitative change over time, students should be able to say that the time is one year and the change is Taylor growing taller.

An example of a quantitative question students should be able to answer is, “Taylor is 3 inches taller this year than last year. Identify the type of change over time.” As students analyze this quantitative change over time, students should be able to say that the time is one year and the change is three inches taller.

Other examples that could be used would be popsicles melting or being eaten, pencils used during the day, or Twizzlers being eaten.

**Assessment Guidelines**

The objective of this indicator is to analyze, which is in the “analyze conceptual” knowledge of the Revised Taxonomy table. To analyze is to break down material (change) into its parts (quality or quantity) and determine how the parts relate to one another and the overall structure. The learning progression to analyze requires students to explore a variety of situations and generate conjectures about the changes their observations (2-1.2). They determine if change has occurred and if so, decide if it is a change in quality or quantity. Students exchange these mathematical ideas (2-1.2) with their classmates and teacher using a variety of forms of communication (2-1.6).
**Standard 2-4:** The student will demonstrate through the mathematical processes an understanding of basic spatial reasoning and the connection between the identification of basic attributes and the classification of three-dimensional shapes.

**Indicator 2-4.1**

Analyze the three-dimensional shapes spheres, cubes, cylinders, prisms, pyramids, and cones according to the number and shape of the faces, edges, corners, and bases of each.

**Continuum of Knowledge**

In kindergarten, students identified two-dimensional shapes: square, circle, triangle, and rectangle as well as the three-dimensional shapes cube, sphere, and cylinder (K-4.1). In addition, students represent two-dimensional shapes (K-4.2). In first grade, students analyzed two-dimensional shapes square, circle, triangle and rectangle (1-4.2). Students classified these two-dimensional shapes as polygons or nonpolygons (1-4.3) and identified three-dimensional shapes prisms, pyramids, and cones (1-4.1).

In second grade, students analyze the three-dimensional shapes: spheres, cubes, cylinders, prisms, pyramids, and cones according to the number and shape of the faces, edges, corners, and bases of each (2-4.1).

In fourth grade, students represent the two dimensional shapes trapezoids, rhombuses and parallelograms and the three dimensional shapes cubes, rectangular prisms, and cylinders (4-4.4).

**Taxonomy Level**

Cognitive Dimension: Analyze  
Knowledge Dimension: Conceptual

**Key Concepts**

- Three-dimensional  
- Shape  
- Sphere  
- Cube  
- Cylinder  
- Prism  
- Pyramid  
- Square pyramid  
- Triangular pyramid  
- Cone  
- Face
Geometry
Second Grade

- Edge
- Corner
- Base
- Horizontal
- Vertical
- Geometry
- Horizontal
- Vertical

**Instructional Guidelines**

For this indicator, it is **essential** for students to:

- Focus more on the properties of figures rather than on simple identification.
- Apply ideas to entire classes of figures, for example, all rectangles, rather than on individual models.
- Analyze classes of figures to determine new properties.
- Recognize the three-dimensional shapes: spheres, cubes, cylinders, prisms, pyramids, and cones including a square pyramid and a triangular pyramid.
- Recognize and define: faces, edges, vertices/corners, and bases.
- Recognize the two-dimensional shapes that make up the three-dimensional shapes.
- Recognize how the number of faces, edges, and corners of the various shapes relate to each other.
- Use proper mathematical vocabulary when referring to the vertices (corners) and horizontal/vertical when referring to position.

For this indicator, it is **not essential** for students to:

None noted

**Student Misconceptions/Errors**

None noted

**Instructional Resources and Strategies**

**Shape Sorts with 3-D Figures**

Have students work in groups of four with a set of 3-D shapes. (Refer to the vocabulary list for the shapes that meet this indicator.)

- **Activity One:** Each child randomly selects a shape. In turn, the students tell one or two things they find interesting about their shape. There are no right or wrong responses.

- **Activity Two:** Children randomly select two shapes. The task is to find something that is alike about their two shapes and something that is different. Have students select their shapes before they know the task.
Geometry
Second Grade

- Activity Three: The group selects one shape at random and places it in the center of the workspace. Their task is to find all other shapes that are like the target shape, but all according to the same rule. For example, if they say, “This one is like our shape because it has a curved side and a straight side,” then all other shapes that they put in the collection must have these properties. Challenge them to do a second sort with the same target shape but using a different property.

- Activity Four: Do a “secret sort”. You or one of the students creates a small collection of about five shapes that fit a secret rule. Leave others that belong in your group in the pile. The other students try to find additional pieces that belong to the set and/or guess the secret rule.

This activity was taken from: Teaching Student-Centered Mathematics, Grades K-3, 2006 edition, pages 194-195, by John Van de Walle.

The Village of Round and Square Houses

Assessment Guidelines

The objective of this indicator is to analyze, which is in the “analyze conceptual knowledge” cell of the Revised Taxonomy table. Conceptual knowledge is not bound by specific examples; therefore, students analyze the three-dimensional shapes: spheres, cubes, cylinders, prisms, pyramids, and cones according to the number and shape of the faces, edges, corners, and bases of each. The learning progression to analyze requires students to recognize the three-dimensional shapes and identify the faces, edges, corners, and bases of each. Students explain and justify their mathematical thinking (2-1.3).
Standard 2-4: The student will demonstrate through the mathematical processes an understanding of basic spatial reasoning and the connection between the identification of basic attributes and the classification of three-dimensional shapes.

Indicator 2-4.2
Identify multiple lines of symmetry.

Continuum of Knowledge
In kindergarten, symmetry is not addressed. In first grade, students examine the two-dimensional shapes: square, triangle, and rectangle to determine if they have a line of symmetry as well as houses, animals, etc. (1-4.4).

In second grade, students identify multiple lines of symmetry in one object including squares, rectangles, triangles, and circles (2-4.2).

In third grade, symmetry is not addressed. In fifth grade, students analyze shapes to determine line symmetry and/or rotational symmetry (5-4.6).

Taxonomy Level
Cognitive Dimension: Remember
Knowledge Dimension: Conceptual

Key Concepts
- Line
- Symmetry
- Multiple
- Identify
- Vertical
- Horizontal
- Diagonal
- Vertices
- Square
- Rectangle
- Triangle
- Circle

Instructional Guidelines
For this indicator, it is essential for students to:
- Understand symmetry.
• Use vocabulary such as vertical, horizontal, diagonal, and vertices when identifying a line of symmetry.

For this indicator, it is **not essential** for students to:

• Include a circle due to its infinite lines of symmetry

**Student Misconceptions/Errors**

None noted

**Instructional Resources and Strategies**

**Pattern Block Symmetry**

Students need a plain sheet of paper with a straight line through the middle. Using about six to eight pattern blocks, students make a design completely on one side of the line that touches the line in some way. The task is to make the mirror image of their design on the other side of the line. When finished, they use a mirror to check their work. They place the mirror on the line and look into it from the side of the original design. With the mirror in place, they should see exactly the same image as they see when they lift the mirror. You can also challenge them to make designs with more than one line of symmetry.


**Folding Shapes**

This activity is taken from **NCTM Navigating through Geometry in Prekindergarten-Grade 2, 2001 edition, page 59.**

**Symmetry and Alphabet Symmetry**

This activity is taken from **Hands-On Math for Grades 2-3, from Creative Teaching Press, 1995 edition, page 99.**

**Assessment Guidelines**

The objective of this indicator is to **identify**, which is in the “remember conceptual knowledge” cell of the Revised Taxonomy table. Conceptual knowledge is not bound by specific examples; therefore, students identify lines of symmetry using a variety of examples. The learning progression to **remember** requires students to recall the meaning of terms such as horizontal, vertical, diagonal, etc.. Students **explore** concrete experiences with squares, rectangles, triangles, and circles in finding lines of symmetry. They **analyze** patterns (2-1.4) and **generate** conjectures (2-1.2) about where the line of symmetry occurs. Students should **explain** and **justify** their mathematical thinking during and after these experiences (2-1.3).
Standard 2-4: The student will demonstrate through the mathematical processes an understanding of basic spatial reasoning and the connection between the identification of basic attributes and the classification of three-dimensional shapes.

Indicator 2-4.3

Predict the results of combining and subdividing polygons and circles.

Continuum of Knowledge

In kindergarten, students identified (K-4.1) and represented (K-4.2) two dimensional shapes: squares, triangles, and rectangles, and circles (and). In first grade, students identified three-dimensional geometric shapes: prism, pyramid, and cone (1-4.1), analyzed two-dimensional shapes: circle, square, triangle, and rectangle (1-4.2), and classified two-dimensional shapes as polygons and non-polynomials (1-4.3).

In second grade, students predict the results of combining and subdividing polygons and circles (2-4.3).

In third grade, the students analyze the results of combining and subdividing circles, triangles, quadrilaterals, pentagons, hexagons, and octagons (3-4.7).

Taxonomy Level

Cognitive Dimension: Understand  
Knowledge Dimension: Conceptual

Key Concepts

- Polygon
- Square
- Triangle
- Rectangle
- Circle
- Combine
- Subdivide
- Predict
- Results
- Symmetry
Instructional Guidelines

For this indicator, it is **essential** for students to:

- Use the two-dimensional shapes with which they have worked to date (circle, square, triangle, and rectangle).
- Use spatial reasoning to visualize what shape might result by combining or subdividing.
- Use their understanding of symmetry to visualize what shape might result by combining or subdividing.

For this indicator, it is **not essential** for students to:

- Use polygons other than a circle, a square, a triangle, or a rectangle.

Student Misconceptions/Errors

None noted

Instructional Resources and Strategies

- Develop their understanding of this skill through numerous and varied experiences with concrete materials such as pattern blocks and computer models in order to develop and expand spatial reasoning.

Assessment Guidelines

The objective of this indicator is to **predict**, which is in the “understand conceptual knowledge” cell of the Revised Taxonomy table. To **predict** is to draw a conclusion from presented information. The learning progression to **predict** requires students to **recall** and **identify** polygons and circles. Students **explore** how to combine and subdivide shapes using concrete and/or pictorial models. Students **generate** conjectures and **exchange** mathematical ideas about their results (2-1.2). Students **generalize** the connection between fractional parts when combining and subdividing polygons and circles (2-1.7). They **use** their understanding from these experiences to make predictions about other situations.
Standard 2-5: The student will demonstrate through the mathematical processes an understanding of the value of combinations of coins and bills and the measurement of length, weight, time, and temperature.

Indicator 2-5.1

Use a counting procedure to determine the value of a collection of coins and bills.

Continuum of Knowledge

In kindergarten, students identified the value of the coins penny, nickel, dime, and quarter and a dollar (K-5.1). In first grade, students used a counting procedure to determine the value of a collection of coins totaling less than one dollar (1-5.1). Students also had experiences representing a nickel, a dime, a quarter, a half-dollar, and a dollar in combinations of coins (1-5.2). First grade students represented money by using the cent and dollar notation. (1-5.3)

In second grade, students use a counting procedure to determine the value of a collection of coins and bills (2-5.1).

Third grade students make change by using the fewest possible number of coins (3-5.1).

Taxonomy Level

Cognitive Dimension: Apply
Knowledge Dimension: Procedural

Key Concepts

- Penny
- Nickel
- Dime
- Quarter
- Half-dollar
- Dollar
- Cents
- Change
- Value
- Coins
- Bills

Instructional Guidelines

For this indicator, it is essential for students to:
Measurement  
Second Grade

- recognize and identify the value of a penny, nickel, dime, quarter, half-dollar, and dollar bill  
- use a counting procedure to determine the value of a collection of coins and bills  
- know how to represent money using the cent and dollar notation  
- know how to skip count by 5’s, 10’s, 25’s

For this indicator, it is **not essential** for students to:

- make change with coins and bills

**Student Misconceptions/Errors**

- Students have a tendency to identify the value of a nickel as 25 cents and a quarter as 5 cents.  
- Students may believe that smaller coins have less value. For example, dimes would be worth less than a nickel because a dime is smaller in size than a nickel.

**Instructional Resources and Strategies**

- Provide many opportunities for students to identify and count collections of coins using real coins or models of real coins.


- Money Skip Counts activity 12.32 (Van de Walle 2007) Explain to the students that they will start skip counting by one number and at your signal they will shift to a count by a different number. Begin with only two different amounts, say, 25 and 10. Write these numbers on the board. Point to the larger number (25), and have students begin to skip-count. After three or more counts, raise your hand to indicate a pause in the counting. Then lower your hand and point to the smaller number (10). Children continue the skip count from where they left off but now count by 10s. Use any two of these numbers: 100, 50, 25, 10, 5, and 1. Always start with the larger. Later, try three numbers, still in descending order.

**Assessment Guidelines**

The objective in this indicator is to use which is in the “apply procedural knowledge” cell of the Revised Taxonomy table. Procedural knowledge is using a specific set of steps and knowing when to use appropriate procedures. The learning progression to use requires students to recognize the coins and recall the value of each coin. Students should also recall their prior knowledge of determining the value of a collection of coins only. As students analyze a collection of coins and bills, they
Measurement
Second Grade

should *generate* conjectures (2-1.2) about how to determine the value of the collection and *exchange* these ideas (2-1.2) with their classmates. Students should be able to determine the value of a collection of coins using manipulatives (concrete) and picture models. Students should also be able to *explain* and *justify* their answers (2.1-3) using appropriate mathematical language.
Standard 2-5: The student will demonstrate through the mathematical processes an understanding of the value of combinations of coins and bills and the measurement of length, weight, time, and temperature.

Indicator 2-5.2

Use coins to make change up to a dollar.

Continuum of Knowledge

At the Kindergarten level, making change is not addresses. First grade students used a counting procedure to determine the value of a collection of pennies, nickels, dimes and quarters totaling less than one dollar (1-5.1). Students also had experiences representing a nickel, a dime, a quarter, a half-dollar, and a dollar in combinations of coins. (1-5.2) First grade students represented money by using the cents and dollar notation. (1-5.3)

In second grade, students will use a counting procedure to determine the value of a collection of coins and bills (2-5.1). Students will also use coins to make change up to a dollar. (2-5.2).

In third grade, students change using the fewest number of coins possible ( 3-5.1).

Taxonomy Level

Cognitive Dimension: Apply
Knowledge Dimension: Conceptual

Key Concepts

- Penny
- Nickel
- Dime
- Quarter
- Half-dollar
- Dollar
- Cents
- Change
- Value

Instructional Guidelines

For this indicator, it is essential for students to:

- identify a penny, nickel, dime, quarter, and half-dollar
- recall the value of a penny, nickel, dime, quarter and half-dollar
• use a counting procedure to determine the value of a collection of coins and bills
• know how to represent money using the cent and dollar notation

For this indicator, it is **not essential** for students to:

• make change using the fewest coins possible at this grade level.
• make change with amounts over $1.00.

**Student Misconceptions/Errors**

• Students may confuse nickels and quarters and the values of each.
• Students may think the size of a coin is related to its value. For example, a nickel is would be worth more than a dime because it is larger in size.

**Instructional Resources and Strategies**

• Students must have numerous opportunities to make change using manipulatives before proceeding to identifying coins needed to make change using picture models.

• How Much More with Coin Numbers? (Van de Walle activity 5.31) At the top of the board, write the values of the coins: 25, 10, 5, and 1. Include 50 if half-dollars are in your curriculum. Write four start and target numbers as in “How Much More?” In this task, however, students must use only numbers in the list to create the difference. They should write down each number they use as they use it. Furthermore, they should try to use as many of the larger numbers as possible, or, in other words, as few “coins” as possible. For example, if the target is 75 with a start of 58, they would write 1, 1, 10, 5. When students share their solutions, do not criticize those who do not use a minimal number of “coins.” Rather, always ask if anyone can do it with fewer “coins”. Ask this even when the solution is the desired one.

• Literature Connection: Alexander Who Used to Be Rich Last Sunday by Judith Viorst Alexander’s grandparents give him a dollar and he spends it on a variety of things.

**Assessment Guidelines**

The objective in this indicator is to use which is in the “apply conceptual knowledge” cell of the Revised Taxonomy table. Conceptual knowledge is the interrelationship between basic elements (individual coins) within a larger structure (coins that can be used to make change in a given situation) and how these function together. The learning progression to use requires students to recognize the coins and recall the value of each coin. Students should also recall their prior knowledge of determining the value of a collection of coins only. As students analyze a situation where change is given, students should generate conjectures (2-1.2) about how to
determine the coins to be used when change is given up to one dollar and exchange these ideas (2-1.2) with their classmates. Students should be able to determine which coins should be used when making change up to one dollar using manipulatives (concrete) and picture models. Students should also be able to explain and justify their answers (2.1-3) using appropriate mathematical language.
**Standard 2-5:** The student will demonstrate through the mathematical processes an understanding of the value of combinations of coins and bills and the measurement of length, weight, time, and temperatures.

**Indicator 2-5.3**

Use appropriate tools to measure objects to the nearest whole unit: measuring length in centimeters, feet, and yards; measuring liquid volume in cups, quarts, and gallons; measuring weight in ounces and pounds; and measuring temperature on Celsius and Fahrenheit thermometers.

**Continuum of Knowledge**

Kindergarten students use nonstandard units to explore the measurement concepts of length and weight. (K-5.3). In first grade, students use whole inch units to measure the length of an object (1-5.4).

Second grade students use appropriate tools to measure objects to the nearest whole unit measuring length in centimeters, feet, and yards; measuring liquid volume in cups, quarts, and gallons; measuring weight in ounces and pounds; and measuring temperature on Celsius and Fahrenheit thermometers. (2-5.3)

Third grade students use appropriate tools to measure objects to the nearest unit: measuring length in meters and half inches; measuring liquid volume in fluid ounces pints, and liters; and measuring mass in grams. (3-5.2)

**Taxonomy Level**

Cognitive Dimension: Apply
Knowledge Dimension: Procedural

**Key Concepts**

- Measure
- Unit
- Ruler
- Centimeter
- Feet
- Yard
- Liquid volume
- Cup
- Quart
- Gallon
- Weigh
- Weight
Measurement
Second Grade

- Ounces
- Pounds
- Temperature
- Thermometer
- Fahrenheit
- Celsius
- Degrees
- Length

Instructional Guidelines

For this indicator, it is **essential** for students to:
- identify appropriate measuring tools to use for a given situation
- know how to measure objects to a whole unit
- measure temperature using thermometers that progress by increments of one
- measure liquid volume using cups, quarts, gallons
- measure weight using ounces and pounds
- measure length using feet, yards, and centimeters

For this indicator, it is **not essential** for students to:
- convert units of measurement
- measure to fractional parts of units

Student Misconceptions/Errors

- Students may think that all objects measured with a ruler must be lined up with the end of the ruler on the 1 rather than at the 0.
- Students may confuse centimeters and inches thinking these are equivalent.
- Students may focus on the end number when measuring an object rather than the number of units. Example: broken ruler

Instructional Resources and Strategies

Second grade is the first time students are introduced to the metric system. Sufficient practice and varied experiences with metric rulers marked in whole centimeter units, customary rulers, and yardsticks will be needed for conceptual understanding of centimeters, feet, and yards. Also, lots of varied experiences measuring weight and temperature with scales and thermometers are necessary as well. Students should have lots of “hands on” experiences measuring with cups, quarts, and gallons. These experiences are needed for conceptual development before moving on to identifying measurements using picture models.
Literature Connections:

- **Hershey’s Milk Chocolate Weights and Measures** by Jerry Pallotta. This book discusses units of measurement for length, weight, and capacity. Tools for measuring are also introduced. The book measures different types of Hershey’s candies that would be familiar to students.
- **Measuring Penny** by Loreen Leedy. This book explores measuring length, weight, capacity, and temperature through a story about caring for a pet dog.

**Assessment Guidelines**

The objective in this indicator is to use which is in the “apply procedural knowledge” cell of the Revised Taxonomy table. Procedural knowledge is bound by specific examples, therefore, students should be able to use appropriate tools to measure objects to the nearest whole unit. The learning progression to use requires students to recognize the appropriate tools for measuring length, liquid volume, weight, and temperature and to explain and justify their reasoning. (2-1.3) Students will then use the appropriate tools to measure objects to the nearest whole unit making real world connections (2-1.7).
Standard 2-5: The student will demonstrate through the mathematical processes an understanding of the value of combinations of coins and bills and the measurement of length, weight, time, and temperature.

Indicator 2-5.4:

Generate common measurement referents for feet, yards, and centimeters.

Continuum of Knowledge:

In first grade, students generate common referents for whole inches. (1-5.5).

In second grade, students should develop their own personal benchmarks for feet, yards, and centimeters and progress to common benchmarks for the class (2-5.4).

In third grade, students should use their understanding of the relationship between meters/yards, kilometers/miles, liters/quarts, and kilograms/pounds to generate common referents (3-5.4).

Taxonomy Level

Cognitive Dimension: Create
Knowledge Dimension: Conceptual

Key Concepts

- Measure
- Measurement
- Feet
- Yard
- Centimeters
- Benchmark

Instructional Guidelines

For this indicator, it is essential for students to:

- Develop a familiarity with the standard units of a foot, yard, and centimeter
- Understand that each one of their common referent may be different

It is not essential for students to:

- use common referents to make estimates
Student Misconceptions:/Errors

- Students might think that a common referent is equivalent to the actual measurement

Instructional Resources and Strategies

- Provide students with many opportunities to create and use personal benchmarks in class
- Create a class list of different personal benchmarks for a foot, yard, and centimeter that students can refer to when measuring
- How Big Is A Foot by Rolf Myller. This book tells a story about using someone’s foot to measure the length of a bed and the problems that occur when measuring using a nonstandard unit.
- About One Unit activity (Van de Walle activity 8.17) For this activity, give students a model of a standard unit and have them search for things that measure about the same as that one unit. For example, to develop familiarity with the meter, give students a piece of rope 1 meter long. Have students make lists of things that are about 1 meter. Keep separate lists for things that are a little less (or more) or twice as long (or half as long). Encourage students to find familiar items in their daily lives. In the case of lengths, be sure to include circular items. Later, students can try to predict if a given object is more than, less than, or close to 1 meter.
- Personal Benchmark activity (Van de Walle activity 8.19) Have students find personal references on their bodies for the following lengths: metric 1cm, 10cm, 1 meter and customary: 1 inch, 1 foot, 1 yard. Have students consider their feet and hands, including lengths and widths of fingers, hands, and hand spans. Heights to waist, shoulder, or head can be considered as well as lengths of arms and arm spans. Students should also have a good idea of how tall they are, their arm span, and how far they normally walk in five or ten strides. After developing lists of benchmarks, have students use only their bodies to “measure” or estimate measures of various lengths and heights. After recording these estimates, they should measure each length with a ruler and check their accuracy.

Assessment Guidelines

The objective of this indicator is to generate which is in the “create conceptual” knowledge of the Revised Taxonomy table. Conceptual knowledge is not bound by specific examples; therefore students should generate common measurement referents for feet, yards, and centimeters. The learning progression to generate requires students to differentiate between feet, yards, and centimeters. Students generate conjectures and exchange mathematical ideas to produce common measurement referent (2-1.2). Throughout this process, students should generalize connections among mathematics and the real world. (2-1.7)
Standard 2-5: The student will demonstrate through the mathematical processes an understanding of the value of combinations of coins and bills and the measurement of length, weight, time, and temperature.

Indicator 2-5.5

Use common measurement referents to make estimates in feet, yards, and centimeters.

Continuum of Knowledge:

In first grade, students were allowed to determine and share their own personal benchmarks. (1-5.5) and use these benchmarks to estimate length to the nearest whole inch. (1-5.6)

Second graders use common measurement referents to make estimates in feet, yards, and centimeters. (2-5.5)

In third grade, students use common referents to make comparisons and estimates associated with meters compared to yards, kilometers to miles, liters to quarts, and kilograms to pounds. (3-5.4)

Taxonomy Level

Cognitive Dimension: Apply
Knowledge Dimension: Procedural

Key Concepts

- Estimate
- Referents
- Feet
- Yards
- Centimeters
- Benchmarks

Instructional Guidelines

For this indicator, it is essential for students to:

- use common referents for feet, yards, and centimeters
- make distinctions between metric and customary units of feet, yards and centimeters

It is not essential for students to:

- make comparisons of units other than feet, yards and centimeters
Student Misconceptions:/Errors

- Students may confuse common referents for inches and centimeters.

Instructional Resources and Strategies

- Second grade is the first time students are introduced to the metric system. Sufficient practice and varied experiences with metric rulers marked in whole centimeter units will be needed for conceptual understanding of centimeters.
- Students will need lots of varied practice and experience using common referents to make estimates using feet, yards, and centimeters. Estimation helps develop familiarity with the standard units.
- Emphasize the use of approximate language in measuring activities. For example, using words such as about, is a little more than, is a little less than can be useful with younger students because many measurements do not come out even. (Van de Walle 2007 p.378)
- Measurement estimation should be an ongoing activity. Having a daily measurement to estimate posted on a bulletin board or chart is one way to provide practice for students.

Assessment Guidelines

The objective of this indicator is to use which is in the “apply procedural knowledge” cell of the Revised Taxonomy table. Procedural knowledge is bound by specific examples. Therefore, students should use common measurement referents to make estimates in feet, yard, and centimeters. The learning progression to use requires students to generate common measurement referents for feet, yards, and centimeters. Students will apply these common referents to make estimates in feet, yard, and centimeters. Students should explain and justify their reasoning when making these estimates (2-1.3) and use a variety of forms of mathematical communication to share their estimates (2-1.6).
Standard 2-5: The student will demonstrate through the mathematical processes an understanding of the value of combinations of coins and bills and the measurement of length, weight, time, and temperature.

Indicator 2-5.6

Predict whether the measurement will be greater or smaller when different units are used to measure the same object.

Continuum of Knowledge:

In Kindergarten and first grade, this standard is not addressed.

In second grade, students predict whether the measurement will be greater or smaller when different units are used to measure the same object. (2-5.6)

This lays the foundation for conversions that will come at later grades (4-5.8).

Taxonomy Level

Cognitive Dimension: Understand
Knowledge Dimension: Conceptual

Key Concepts

- Predict
- Measurement
- Greater
- Smaller

Instructional Guidelines

For this indicator, it is essential for students to:

- know inches are smaller units than feet and feet are smaller units than yards
- understand that larger units will create a smaller measure and smaller units will create a larger measure

It is not essential for students to:

- convert units of measurement
Measurement
Second Grade

Student Misconceptions:/Errors

- Students have difficulty understanding that the smaller the unit the larger the measure and the larger the unit the smaller the measure.

Instructional Resources and Strategies

Students need numerous experiences predicting whether the linear measurement will be greater or smaller when different units are used to measure the same object. The emphasis is on conceptual development and understanding. Students need to test predictions by measuring objects using different units.

The Changing Units activity (Van de Walle 2007 p. 380) is an activity that can provide practice where this skill is the focus. Have students measure a length with a specified unit. Then provide them with a different unit that is either twice as long or half as long as the original unit. Have students predict the measure of the same length using the new unit. Students should write down their predictions and explain how these were made. Discuss predictions and then have them make the actual measurement. Cuisenaire rods are good to use for this activity.

Assessment Guidelines

The objective of this indicator is to predict which is in the “understand conceptual knowledge” cell of the Revised Taxonomy table. Conceptual knowledge is not bound by specific examples. Students predict whether measurements will be greater or smaller when different units are used to measure the same object. The learning progression to predict requires students to recall the size of a centimeter, inch, and foot and compare these units. Students generalize math concepts such as the smaller the unit used to measure, the larger the measure and the larger the unit the smaller the measure (2-1.5). Students should explain and justify their predictions. (2-1.3)
Standard 2-5: The student will demonstrate through the mathematical processes an understanding of the value of combinations of coins and bills and the measurement of length, weight, time, and temperature.

Indicator 2-5.7

Use analog and digital clocks to tell and record time to the nearest quarter hour and to the nearest five-minute interval.

Continuum of Knowledge:

Kindergarten students used analog and digital clocks to tell time to the hour. (K-5.6) First grade students used analog and digital clocks to tell and record time to the hour and half-hour. (1-5.8)

In second grade, students use analog clocks to tell and record time to the nearest quarter hour and to the nearest five minute interval. (2-5.7)

Third grade students will use analog and digital clocks to tell time to the nearest minute. (3-5.5)

Taxonomy Level

Cognitive Dimension: Apply
Knowledge Dimension: Procedural

Key Concepts

- Analog
- Digital
- Time
- Clock
- Hour
- Half-hour
- Quarter hour
- Minute
- Interval

Instructional Guidelines

For this indicator, it is essential for students to:

- recognize and name numbers 1-60, using skip counting by fives
- write the numbers 1-60
- count by fives and ones
• identify locations of the minute hand at quarter hours and five minute
  intervals

It is **not essential** for students to:

• tell time to the nearest minute
• write the time in words

**Student Misconceptions/Errors**

• Students often write five minutes past the hour as a one digit number
  instead of a two digit number. Example 7:5 instead of 7:05

**Instructional Resources and Strategies**

Students should have lots of experiences using student clocks to show/illustrate
time to five minute intervals and quarter hours before proceeding to the use of
pictorial models of clocks. The following suggestions can help students understand
and read analog clocks.

1. Begin with a one-handed clock. A clock with only an hour hand can be read
   with reasonable accuracy. Use lots of approximate language. “It’s about 7
   o’clock.” “It’s a little past 9 o’clock.” “It’s halfway between 2 o’clock and 3
   o’clock.”

2. Discuss what happens to the big hand as the little hand goes from one hour
to the next. When the big hand is at 12, the hour hand is pointing exactly to
   a number. If the hour hand is about halfway between numbers, about where
   would the minute hand be? If the hour hand is a little past or before an hour
   (10 to 15 minutes) about where would the minute hand be?

3. Use two real clocks, one with only an hour hand and one with two hands. (Break
   off the minute hand from an old clock.) Cover the two-handed clock.
   Periodically during the day, direct attention to the one-handed clock. Discuss
   the time in approximate language. Have students predict where the minute
   hand should be. Uncover the other clock and check.

4. Teach time after the hour in 5-minute intervals. After step 3 has begun,
count by fives going around the clock. Instead of predicting that the minute
hand is pointing at the 4, encourage students to say it si about 20 minutes
after the hour. As skills develop, suggest that students always look first at
the little or hour hand to learn approximately what time it is and then focus
on the minute hand for precision.

5. Predict the reading on a digital clock when shown an analog clock, and vice
versa; set an analog clock when shown a digital clock. This can be done with
both one-handed and two-handed clocks.

**Assessment Guidelines**

The objective of this indicator is to **use** which is in the “apply procedural knowledge”
cell of the Revised Taxonomy table. Procedural knowledge is bound by specific
examples, therefore students **tell** and **record** time to the nearest quarter hour and
five minute intervals using analog and digital clocks. The learning progression to apply requires that students recognize time to the hour and half hour and recall skip counting by 5’s. Students also analyze patterns on the analog clock (2-1.4) and generalize connections in mathematics between skip counting and telling time. (2-1.7) Students use multiple representations to convey time (2-1.8). For example, 2:15 is also quarter past 2 or 15 minutes after 2.
Standard 2-5: The student will demonstrate through the mathematical processes an understanding of the value of combinations of coins and bills and the measurement of length, weight, time, and temperature.

Indicator 2-5.8

Match a.m. and p.m. to familiar situations.

Continuum of Knowledge:

This standard is not addressed at the Kindergarten or first grade level.

Second grade students will match a.m. and p.m. to familiar situations. They also use analog and digital clocks to tell and record time to the nearest quarter hour and to the nearest five-minute interval (2-5.7).

In third grade, students use analog and digital clocks to tell time to the nearest minute (3-5.6).

Taxonomy Level

Cognitive Dimension: Factual
Knowledge Dimension: Understand

Key Concepts

- A.M./ a.m.
- P.M./ p.m.

Instructional Guidelines

For this indicator, it is essential for students to:

- Understand the time frame represented by a.m.
- Understand the time frame represented by p.m.
- Relate a.m. and p.m. to familiar situations

For this indicator, it is not essential for students to:

None noted

Student Misconceptions/Errors

- Students confuse midnight as 12:00 p.m. instead of a.m. because it is nighttime
- Students think a.m. always refers to daytime and p.m. as nighttime.
Instructional Resources and Strategies

- Second grade is the first time the concepts of a.m. and p.m. are introduced. Students need to relate these to familiar situations. (2-5.8) Students should have experience with both the a.m. and p.m. and the A.M. and P.M. forms of the abbreviations. Students need lots of practice relating a.m. and p.m. to familiar situations.
- Having students keep a 24 hour timeline can help students gain an understanding of this concept.
- Talk with students about familiar a.m. and p.m. situations, such as the sun comes up in the morning(a.m.) and sets in the evening(p.m.); we eat breakfast in the morning(a.m.) and eat dinner in the evening(p.m.).
- Have discussions about noon as the beginning of p.m.; if you have lunch before noon (12:00 p.m.), it is a.m., if you eat at noon or after noon, it is p.m.

Assessment Guidelines

The objective of this indicator is to match which is in the “factual understand” cell of the Revised Taxonomy table. Factual knowledge is bound by specific examples. Students should match a.m. and p.m. to familiar situations. The learning progression to match requires students to interpret differences between a.m. and p.m. Students will apply this knowledge to familiar situations and explain and justify their answers. (2-1.3)
Standard 2-5: The student will demonstrate through the mathematical processes an understanding of the value of combinations of coins and bills and the measurement of length, weight, time, and temperature.

Indicator 2-5.9

Recall equivalencies associated with length and time: 12 incher = 1 foot, 3 feet = 1 yard, 60 minutes = 1 hour, and 24 hours = 1 day

Continuum of Knowledge:

In kindergarten, students learned to recall the equivalencies 7 days = 1 week and 12 months = 1 year (K-5.8).

Second grade students recall the equivalencies 12 inches = 1 foot, 3 feet = 1 yard, 60 minutes = 1 hour, and 24 hours = 1 day (2-5.9).

In third grade, students will learn to recall the equivalency 60 seconds = 1 minute (3-5.7).

Taxonomy Level

Cognitive Dimension: Recall
Knowledge Dimension: Factual

Key Concepts

- Minute
- Hour
- Day
- Inches
- Foot
- Feet
- Yard
- Equivalent

Instructional Guidelines

For this indicator, it is **essential** for students to:

- know the equivalencies 12 inches = 1 foot, 3 feet = 1 yard, 60 minutes = 1 hour

For this indicator, it is **not essential** for students to:

- know the equivalency 60 seconds = 1 minute
- convert units of measurement
Student Misconceptions/Errors

None noted

Instructional Resources and Strategies

Students need to recall the listed equivalencies, therefore student experiences should be numerous and varied enough so that they understand the equivalencies. For example, students can take a 12 inch ruler and place on a yardstick to discover that 3 feet = 1 yard.

Assessment Guidelines

The objective of this indicator is to recall which is in the “remember factual” knowledge cell of the Revised Taxonomy table. Although the focus of the indicator is to recall, the learning progression should integrate concept building activities that support retention of these facts. The learning progression to recall equivalencies requires students to explore these equivalencies using concrete and/or pictorials models. As students generalize connections among these models and mathematics (2-1.7), they generate conjectures (2-1.2) about the relationships between these measures. They explain and justify their answers (2-1.3) to their classmates and teacher. Students may use a variety of forms of communication to recall these equivalencies such as using words, pictures, and/or numbers. (2-1.6)
**Standard 2-6:** The student will demonstrate through the mathematical processes an understanding of creating questions to collect data, organizing data, describing trends of a data set, and making predictions based on data.

**Indicator 2-6.1**

Create survey questions to collect data.

**Continuum of Knowledge**

In first grade, students used survey questions to collect data. (1-6.1)

In second grade, students create survey questions to collect data. (2-6.1)

In fourth grade, students compare how data-collection method impact survey results (4-6.1).

**Taxonomy Level**

Cognitive Dimension: Create

Knowledge Dimension: Conceptual

**Key Concepts**

- Survey
- Data

**Instructional Guidelines**

For this indicator, it is *essential* for students to

- create survey questions to collect data.

For this indicator, it is *not essential* for students to

None noted

**Student Misconceptions/Errors**

None noted

**Instructional Resources and Strategies**

The teacher must model the steps in creating a survey question, and then provide time for students to create their own questions and collect their data. Students should be writing questions that could be used to interpret the data as introduced in first grade. When students formulate the questions they want to ask, the data
becomes more and more meaningful. They then have a purpose for organizing the data.

Assessment Guidelines

The objective of this indicator is to create, which is in the “create conceptual” knowledge of the Revised Taxonomy table. Conceptual knowledge is not bound by specific examples; therefore, students create survey questions to collect data. The learning progression to create requires students to understand the reasonableness of a good survey question. For example, “Did you drive to school today?” is not an appropriate survey question for second graders. Students generate conjectures (survey questions) and exchange mathematical ideas as to the reasonableness of their questions (2-1.2). Students generalize connections among mathematics, the environment, and other subjects (2-1.7).
Standard 2-6: The student will demonstrate through the mathematical processes an understanding of creating questions to collect data, organizing data, describing trends of a data set, and making predictions based on data.

Indicator 2-6.2

Organize data in charts, pictographs, and tables.

Continuum of Knowledge

In kindergarten, students organized data in graphic displays in the form of drawings and displays (K-6.1). In first grade, students organized data in picture graphs, bar graphs, and tables (1-6.2).

In second grade, students organized data in charts, pictographs, and tables. (2-6.2)

In third grade, students organized data in tables, bar graphs, and dot plots. (3-6.2)

Taxonomy Level

Cognitive Dimension: Analyze
Knowledge Dimension: Conceptual

Key Concepts

- organize
- data
- data set
- charts
- pictographs
- value
- tables

Instructional Guidelines

For this indicator, it is essential for students to

- organize data in charts, pictographs, and tables.
- Identify which data sets are best represented by different graph types

For this indicator, it is not essential for students to

- organize data in bar graphs and dot plots.
- use scales greater than one on a pictograph.
Student Misconceptions/Errors

While students will informally use a scale greater than one with their pictographs, students will often interpret the partial picture as one-half rather than a whole number.

Instructional Resources and Strategies

A chart is a table that may have words, diagrams, and/or numbers. A table is an arrangement of rows and columns. A pictograph is similar to a picture graph, but it uses one type of picture to show and compare data. At the bottom of the pictograph is a key that indicates the value of the picture. The teacher must model the reasoning behind choosing the value for the picture in the pictograph. If most of the votes were odd numbers, and the data set was small, the value of the picture should be 1 instead of 2. The larger the data set, the picture should be a larger value so a reasonable number of pictures will fit the pictograph. While students may informally use a scale larger than one it is not a requirement of second grade students. The concept of scales greater than one is introduced in fourth grade. It is imperative that teachers review the graphs introduced in first grade in order to make certain that learning experiences build on that prior knowledge and is not a repeat of prior knowledge. Data collection and analysis should be a frequent activity throughout the school year.

The Investigations Curriculum (in the Mathematical Thinking manual) from Scott Foresman/Pearson Publishing introduces students in grades K-2 to the Question of the Day. This daily activity that may be used to help meet this indicator.

Assessment Guidelines

The objective of this indicator is to organize, which is in the “analyze conceptual” knowledge of the Revised Taxonomy table. To organize is to determine how elements (data) fit or function within a structure (charts, pictographs, and tables). The learning progression to organize requires students to recall and understand the structure of charts, pictographs and tables. Students analyze data that they have collect or have been given to generalize connections (2-1.7) among the data. They use these connections to determine how to best organize the data. Students explain and justify their answers (2-1.3) to their classmates and teacher using a variety of form of communication (2-1.6).
Data Analysis and Probability
Second Grade

**Standard 2-6:** The student will demonstrate through the mathematical processes an understanding of creating questions to collect data, organizing data, describing trends of a data set, and making predictions based on data.

**Indicator 2-6. 3** Infer trends in a data set as increasing, decreasing, or random.

**Continuum of Knowledge**
In kindergarten, students interpreted data from a graph. (K-6.2) In first grade, students interpreted data in picture graphs, object graphs, bar graphs, and tables by using the comparative terms *more, less, greater, fewer, greater than, less than.* (1-6.3)

In second grade, students infer trends in a data set as increasing, decreasing, or random. (2-6.3)

In third grade, students interpret data in tables, bar graphs, pictographs, and dot plots. (3-6.3)

**Taxonomy Level**
Cognitive Dimension: Understand
Knowledge Dimension: Conceptual

**Key Concepts**

**Vocabulary**
Data
Data set
Increasing
Decreasing
Random
Trends in a data set
Infer
Instructional Guidelines
For this indicator, it is essential for students to
• analyze the data set for trends as increasing, decreasing, or random.
• understand differences in trends in data sets.

For this indicator, it is not essential for students to
• interpret data in bar graphs and dot plots.

Misconceptions
None noted

Instructional Resources and Strategies
In first grade, students interpreted the data using the terms more, less, greater, fewer, greater than, and less than. They will now interpret the data and then be able to use those comparisons to infer trends. An example of using data at this level is for students to collect and record the temperature in degrees Fahrenheit at the same time each school day for two weeks. (This activity works best in the late fall or early spring when the temperature fluctuates more dramatically.) After the second week of data collection, ask the students what facts they can state about the measurements: Is there a pattern or trend in the numbers? Are the numbers increasing (getting warmer), decreasing (getting cooler), or are the numbers random and not occurring in any particular pattern?

Assessment Guidelines
The objective of this indicator is to infer, which is in the “understand conceptual” knowledge of the Revised Taxonomy table. Conceptual knowledge is not bound by specific examples; therefore, students infer trends in a data set as increasing, decreasing, or random. The learning progression to infer requires students to collect and represent data in charts, pictographs, and tables. Students generate conjectures and exchange mathematical ideas to infer trends in data sets as increasing, decreasing, or random.
**Standard 2-6:** The student will demonstrate through the mathematical processes an understanding of creating questions to collect data, organizing data, describing trends of a data set, and making predictions based on data.

**Indicator 2-6. 4** Predict on the basis of data whether events are *more likely* or *less likely* to occur.

**Continuum of Knowledge**
In first grade, students predicted on the basis of data whether events are likely or unlikely to occur. *(1-6.4)*

In second grade, students predict on the basis of data whether events are *more likely* or *less likely* to occur. *(2-6.1)*

In third grade, students predict on the basis of data whether events are *likely*, *unlikely*, *certain*, or *impossible* to occur. *(3-6.6)*

**Taxonomy Level**
Cognitive Dimension: Understand
Knowledge Dimension: Conceptual

**Key Concepts**
**Vocabulary**
- Data
- Events
- More likely
- Less likely
- Occur
- Predict

**Instructional Guidelines**
For this indicator, it is **essential** for students to
- predict on the basis of data whether events are more likely or less likely to occur.
- understand the meaning of more and less.
For this indicator, it is **not essential** for students to
- predict on the basis of data whether events are likely, unlikely, certain, or impossible to occur.

**Misconceptions**
Many young children believe that an event will happen “because it is my favorite color” or “because it is lucky” or “because it did it that way last time.”

**Instructional Resources and Strategies**
Probability is more than spinning spinners and flipping coins. It helps us answer questions about our world in terms of the chances of future events occurring or not (Teaching Student Centered Mathematics, Van de Walle, page 387, 2004 edition).

In order for students to truly form a foundation of understanding about the language of these terms, the teacher should present a hands-on activity such as “Empty the Bowl” (Math By All Means: Data and Chance, Grades 1-2, Marilyn Burns) to get students fully engaged in understanding the concepts. Spinners, numeral cubes, etc. are essential materials needed in many probability experiments. These materials make the abstract concepts of most likely and most unlikely concrete for children. Once again, the data collected should be organized in the tables, charts, and graphs; and students should be expected to interpret the data and other trends.

**Assessment Guidelines**
The objective of this indicator is to **predict**, which is in the “understand conceptual” knowledge of the Revised Taxonomy table. Conceptual knowledge is not bound by specific examples; therefore, students **predict** on the basis of the data whether events are more likely or less likely to occur. The learning progression to **predict** requires students to **collect** and **represent** data using a variety of forms of mathematical communication (2-1.6). Students **generate** conjectures and **exchange** mathematical ideas to **predict** whether events are more likely or less likely to occur.